1) (10 points) Among five applicants for chemical engineering position in a firm, two are rated as excellent and the other three are rated as good. A manager randomly chooses two of these applicants to interview. Find the probability that the manager chooses

a) the two rated as excellent.
b) the two rated excellent, given that one of the chosen two is already known to be excellent.

Choosing two from five can be done in C_2^5 ways $= \frac{5!}{2!(3!)} = \frac{4\times5}{2} = 10$ ways

a) $P(2 \text{ excellent}) \cdot C_2^2 C_0^3 = \frac{2!}{2!} \frac{3!}{0!(3!)} = 1$ way

$\Rightarrow P(2 \text{ excellent}) = \frac{1}{10}$

b) $P(\underbrace{2 \text{ excellent}}_{A} \cap \underbrace{\text{excellent}}_{B}) = \frac{P(A \cdot B)}{P(B)}$

7

we will have 1 excellent among 4

$P = \frac{C_1^1}{C_1^4} = \frac{1}{4} = 0.25$

2) (28 points) An accounting firm that does not have its own computing facilities rents time from a consulting company. The firm must plan its computing budget carefully and hence has studied the weekly use of CPU time quite throughly. The weekly use of CPU time approximately follows the probability density function given by (measurements in hours)

$$f(x) = \begin{cases} \frac{3}{64}x^2(4-x) & 0 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

a) Find the distribution function $F(x)$ for weekly CPU time X.

b) Find the expected value and variance of weekly CPU time.

c) The CPU time costs the firm $200 per hour. Find the expected value and variance of the weekly cost for CPU time.

d) In the coming 5 weeks, if we assume that the weekly CPU time is independent from week to week, find the probability that the weekly cost for CPU time will exceed $600 exactly 2 times.

25

a) $F(x) = \int_0^b \frac{3}{64} x^2(4-x) = \frac{3}{64}\int_0^b 4x^2 - x^3$

$= \frac{3}{64}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^b = \frac{1}{16}b^3 - \frac{3b^4}{256}$ $\quad 0 < x < 4$

$= 0 \quad b < 0$

$= 1 \quad b > 4$

b) $E(x) = \int_0^4 x f(x) = \frac{3}{64}\int_0^4 x^3(4-x)$

$= \frac{3}{64}\int_0^4 4x^3 - x^4$

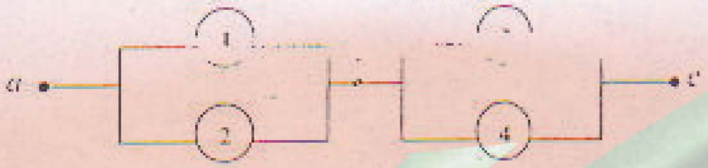
$= \frac{3}{64}\left[x^4 - \frac{x^5}{5}\right]_0^4$

$= \frac{3}{64}[256 - 204.8]$

$= 2.4$

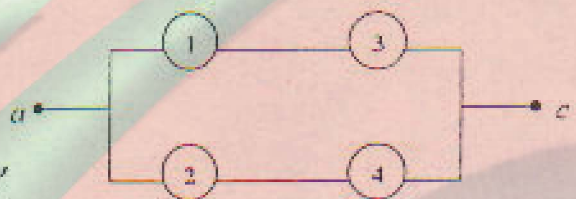
$V(x) = E(x^2) - (E(x))^2$

3) (12 points) Relays in a section of an electrical circuit operate independently, and each one closes properly when a switch is thrown with probability 0.9. the following two designs, each involving four relays are presented for a section of a new circuit.



A

12



B

For each one of these designs, find the probability of current flowing from a to c when the switch is thrown.

design A.

for current flow from a to b is

$1 - p(\text{both closed}) = 1 - (0.1)(0.1) = 0.99$

same for the current from b to c

=> probability for the current to path from A to c is

$p = (0.99)(0.99) = 0.9801$ ✓

design B:

for the above path the probability to pass = $p(\text{both closed}) = 0.81$ => $q_1 = 0.19$

same for the lower connection: $p_2 = 0.81$ $q_2 =$

=> $p(\text{current from a to c}) = 1 - p(\text{both open})$

$= 1 - (0.19)(0.19)$

$= 0.9639$

4) **(10 points)** A type of capacitor has resistances that vary according to a normal distribution with a mean of 800 megohms and a standard deviation of 200 megohms. A certain application specifies capacitors with resistances between 900 and 1000 megohms.

a) What proportion of these capacitors will meet this specification?

b) If two capacitors are randomly chosen from a lot of capacitors of this type, what is the probability that both will satisfy the specification?

$\mu = 800\ M\Omega$

$\delta = 200\ M\Omega$

10

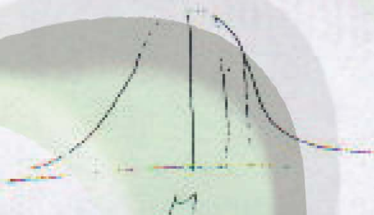
$a = 900$

$b = 1000$

Normal distribution

$z_a = \frac{900-800}{200} = \frac{1}{2}$

$z_b = \frac{1000-800}{200}$



$P(Z \le z_b) = 0.3413$ } $0.3413 - 0.1915 = 0.1498$

$P(Z \le z_a) = 0.1915$

The proportion that meets the specification is

$0.1498 \times 100 \cong 15\%$

b) ~~Binomial case with p = 0.15~~

~~P(1 success) = 0.15~~

Both success

Binomial case n = 2

$p = 0.15 \quad q = 0.85$

$P(X=2) = C^2_2\, p^2 q^0 = (0.15)^2 = .0225$

5) (15 points) A geological study indicates that an exploratory oil well drilled in a certain region should strike oil with probability 0.2.

a) Find the probability that the first strike of oil comes on the third well drilled.

b) Suppose a company wants to set up three producing wells. Find

i) The probability [illegible]

ii) That expected value of the number of wells that must be drilled to find the three successful ones.

What assumptions are necessary for your answer to be correct?

a) Negative Binomial case $p=0.2$ $q=0.8$

$x=3$

$p(x=3) = C^{2}_{0}\, p q^2 = 0.2(0.8)^2 = 0.128$

b) i) $r=3$ $x=5$

$p(x=5) = C^{4}_{2}\, p^3 q^2 = \frac{4!}{2!2!}(0.2)^3(0.8)^2 = 0.03072$

ii) $r=3$

$E(x) = \frac{r}{p} = \frac{3}{0.2} = 15$

we assumed that each drill is independent from the other

6) (25 points) Customer arrivals at a checkout counter in a department store have a poisson distribution, with an average of 6 customes per hour.

a) For a given hour, find the probability that no more than one customer arrive.

b) Find the mean and the probability density function of the waiting time between the opening of the counter and the arrival of the first customer.

c) If a clerk takes approximately 15 minutes to serve the first customer arriving at the counter, what is the probability that at least one more customer is waiting when the service of the first customer is compacted?

d) Find the mean, variance and the probability density function of the waiting time between the opening of the counter and the arrival of the third customer.

$\lambda = 6/hr$

a) $P(X \le 1) = p(x=0) + p(x=1)$ where $\left(p(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}\right)$

20/

$= e^{-6} + \frac{e^{-6}6}{1}$

$= 7e^{-6}$

$= 0.0174$

b) We have a ~~gamma~~ exponential distribution

with mean $\beta = \frac{1}{\lambda} = \frac{1}{6} = 10$ min

and its probability density function is given by

$f(x) = \frac{1}{\beta}e^{-x/\beta} = 6e^{-6x}$ X in hours

or $= \frac{1}{10}e^{-\frac{x}{10}}$ X in minutes.

c) 15 min to serve

at least one customer $\Rightarrow P(10 < X < 15)$ $P(X<15) = 1 - P(X>15)$ minutes

$P = e^{-10/10} - e^{-15/10} = 0.14475$

d) gamma dist with $E(x) = \alpha\beta$ where $\alpha = 3$

$\Rightarrow E(x) = 30$ min !! $E(x^2) = 3 \cdot 4 \cdot 100 = 1200$

$\Rightarrow V(x) = E(x^2) - (E(x))^2 = 1200 - 900 = 300$

Here $E(x^2) = \int x^2 f(x)$

$= \frac{3}{64}\int_0^4 4x^4 - x^5$

$= \frac{3}{64}\left[\frac{4x^5}{5} - \frac{x^6}{6}\right]_0^4$

$= \frac{3}{64}$ 6.4

$V(x) = 6.4 - (2.4)^2 = 0.64$

$C = 200x$

$E(C) = 200E(X)$

$= 200(2.4)$

$= 480\$$

$V(C) = (200)^2 V(X)$

$= 160000V$

exceed 600\$ $\Rightarrow x \geq 3$

$P(x \geq 3) = \int_3^4 \frac{3}{64}x(4-x) = \frac{3}{64}\int_3^4 4x^2 - x^3 = \frac{3}{64}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_3^4$

$= \frac{3}{64}[21.33 - 15.75] = 0.262$

Have a binomial case with $x = 2$, $n = 5$, $p = 0.262$, $q = 0.738$

$P(x=2) = C^5_2 p^2 q^3 = \frac{5!}{2!(3!)}(0.262)^2(0.738)^3$

$= 10(0.0276) = 0.276$